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The Stability of Component Assets in Optimal Portfolios of Stock and Commodity Indexes

Abstract. The turbulences in financial markets increased the interest in commodity investments as an alternative asset class for potential risk diversification. A plethora of past and present studies documents the diversification benefits achieved by adding commodities to the traditional security portfolios. Most of commodity diversification papers ignore the stability of component assets in the optimal portfolio. This paper examines both, the stability and performance of optimal Markowitz portfolios over time. The portfolios are composed of commodity and stock indexes. Their risk and returns are compared to the risk and return of the equally weighted benchmark portfolio.

Key words: stock and commodity indexes, Markowitz portfolio optimization, component stability

Introduction

Modern finance owes its origin to the introduction of Markowitz portfolio theory, which subsequently became a basic tenet of finance. Since Markowitz' seminal paper (Markowitz, 1952) we have known that diversification can increase portfolio expected returns while reducing volatility. Following Markowitz, many researchers worked on his model and extended it by adding a variety of assumptions, constraints or objectives such as cardinality constraint, transaction costs, skewness and kurtosis to make it more realistic, because the criticism on the model is mainly focused on the fact it oversimplifies reality through some of its assumptions, e.g. normally distributed returns or efficient markets (Gasser, Rammenstorfer and Weinmayer, 2017). Mashayekhi and Omrani (2016) provide a detailed review of studies which extended the Markowitz model. According to Zopounidis, Doumpos and Fabozzi (2014), the principles introduced through the model are still at the core of many modern approaches for asset allocation, investment analysis, risk management, capital budgeting, and decision making under uncertainty. Although the complexity, vulnerabilities and the uncertainties involved in the globalized business and financial environments increase, the framework introduced by Markowitz continues to be highly relevant.

Originally, in Markowitz portfolio theory the diversification effect was achieved by the increase in the number of shares in the portfolio. However, changes in international financial markets, e.g. high volatility and contagion risk arising from increased financial integration and interdependence among stock markets have reduced diversification benefits for institutional investors and created higher systemic risk (Bekiros et al., 2016). This has driven some investors and portfolio managers to seek for diversification benefits in other asset classes, such as commodities. According to Nijman and Swinkels (2008), the interest

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in portfolio diversification through commodity investments dates at least back to Bodie (1980) who points out the potential benefits of commodities for pension funds. Froot (1995) suggests that commodities are better portfolio diversifiers than, for example, real estate. Chow et al. (1999) indicate that commodities can be particularly valuable diversifiers in adverse economic circumstances, when other alternative assets tend to correlate more with traditional assets. More recently, Erb and Harvey (2006), Conover et al. (2010), You and Deigler (2012) document the diversification benefits from adding commodities to a stock- or bond-portfolio.

Although it is well known that commodity investment provides diversification benefits to a portfolio, most commodity diversification papers ignore an important issue that is the stability of component assets in the optimal portfolio. That is why the aim of the paper is to examine the stability of the weights of the components in the Markowitz portfolio composed of commodity and stock indexes. The inspiration for the research is the paper by You and Deigler (2012), who employ a wide variety of types of futures contracts to study the potential portfolio benefits of adding individual commodity futures to a traditional portfolio and to study the stability of portfolio's components from year to year. Our investigation focuses on commodity indexes as they offer broad exposure to commodity assets, subsectors and markets. Moreover, they have low correlations to traditional asset classes such as fixed income and equities, so we would expect a diversification benefit by adding an allocation to commodity indexes to a portfolio of bonds and stocks.

Although there are several major investable indexes, with a range of compositions and methodology of construction, the most popular among investors is the Standard & Poor's Goldman Sachs Commodity Index (S&P GSCI) and the Dow-Jones AIG Commodity Index that was rebranded under its current name – the Bloomberg Commodity Index (BCOM) in 2014. In order to provide a comprehensive review that considers a variety of commodity index options, we additionally take into account the Thomson Reuters/Core Commodity CRB Index (TR/CC CRB) and the Deutsche Bank Liquid Commodity Index (DBLCI). The investigation also covers the most important stock indexes: the Financial Times Stock Exchange 100 Index (FTSE 100),the IBOVESPA, the Standard & Poor's 500 (S&P 500), the Nikkei 225, the Shanghai Composite Index (SSE), and the All Ordinaries Index (AOI).

Methodology

In order to set an optimal portfolio composed of commodity and stock indexes we employ two options of the classical Markowitz model:

- option **A** that minimizes the portfolio variance of returns:

$$s_p^2 = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j k_{ij} \tag{1}$$

subject to:

$$\sum_{i=1}^{k} x_i = 1, \tag{2}$$

$$\sum_{i=1}^{k} x_i \bar{z}_i \geq \gamma, \tag{3}$$

$$x_i \geq 0 \quad i = 1, \ldots, n, \tag{4}$$

where:
x_i - weight of the *i*-th asset in the portfolio,
γ - predetermined portfolio return fulfilling the assumption that $\gamma \leq \max \bar{z}_i$,
$\bar{z}_i$ - mean return on the *i*-th asset,
k_{ij} - covariance between the returns on the *i*-th and the *j*-th assets;
- option **B** that maximizes portfolio return:

$$R_p = \sum_{i=1}^{k} x_i \bar{z}_i \tag{5}$$

subject to:

$$\sum_{i=1}^{k} x_i = 1, \tag{6}$$

$$s_p^2 = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j k_{ij} \leq \omega, \tag{7}$$

where:
ω – predetermined portfolio variance of returns fulfilling the assumption that $\omega \leq \max S_i^2$ (S_i^2 – variance of the *i*-th asset).

We also construct the benchmark portfolio which is the traditional portfolio generated by equally weighting the assets (indexes).

Data and preliminary analysis

Empirical data covers daily quotes of four commodity indexes and six stock indexes, representing Australian, North and South American, Asian and European stock markets, from January 5, 2009 to December 30, 2015. As it is mentioned in the introduction, the commodity indexes under consideration are the following:

- the Thomson Reuters/Core Commodity CRB Index,
- the Bloomberg Commodity Index,
- the S&P Goldman Sachs Commodity Index,
- the Deutsche Bank Liquid Commodity Index.

Table 1 presents their main characteristics. In theory, commodity indexes share a similar goal: to create a broad indicator of commodity price movement. In practice, portfolio weightings, construction, and calculation methodology vary dramatically from one index to another. The TR/CC CRBI, originally designed to provide dynamic representation of broad trends in overall commodity prices, has equal weightings for each of the component commodities. The S&P GSCI uses a weighting rule that attempts to capture the rationale behind market-capitalization weighting. It weights each constituent commodity by a dollar estimate of the global production of that commodity. The DLBCI considers both world production and world inventory. The S&P GSCI uses open interest on the futures contract but focuses on economic factors by attempting to filter out purely speculative interest.

The stock indexes under consideration represent stock markets from different continents:

- British Financial Times Stock Exchange 100 Index (FTSE 100) – Europe,
- Brazilian IBOVESPA – South America,
- the U.S. Standard & Poor's 500 (S&P 500) – North America,
- Japanese Nikkei 225 – Asia,
- Chinese Shanghai Composite Index (SSE) – Asia,
- the All Ordinaries Index (AOI) – Australia.

Table 1. Description of commodity indexes

Characteristic	Index name			
	Thomson Reuters/Core Commodity CRB Index	S&P Goldman Sachs Commodity Index	Bloomberg Commodity Index	Deutsche Bank Liquid Commodity Index
Start of back-filled history	1982	1969	1991	1988
Start of investable history	1986	1992	1998	2003
Indexing strategy	Equal-weighted	World-production weighted	Liquidity and dollar-adjusted production-weighted	World production and inventory-weighted

Source: www.vanguard.com.

Figures 1 and 2 show the performance of stock and commodity indexes in the period from January 5, 2009 to December 30, 2015. They let us notice that three out of four commodity indexes reached their lows in December 2015 and all of them reached their highs in April 2011, whereas the majority of stock indexes reached minimal levels in March 2009. Their maximal levels were hit in mid-2015, when commodity indexes, exhibiting their counter-cyclic nature, were falling down.

In tables 2 and 3, there are reported mean logarithmic returns and standard deviations for all commodity and stock indexes under consideration. The results vary from year to year, however the detailed analysis of estimates in table 2 let us conclude that 2009 is the only year when all indexes generate positive mean returns. Taken into account the whole period from 2009 through 2015, almost all commodity indexes are characterized by negative mean returns. The only exception is the Bloomberg Commodity Index (BCOM). The highest positive expected rate of return is that of S&P500 and also all other stock indexes exhibit positive mean returns. However, the S&P 500 is the only index providing positive mean returns in each of considered annual periods.

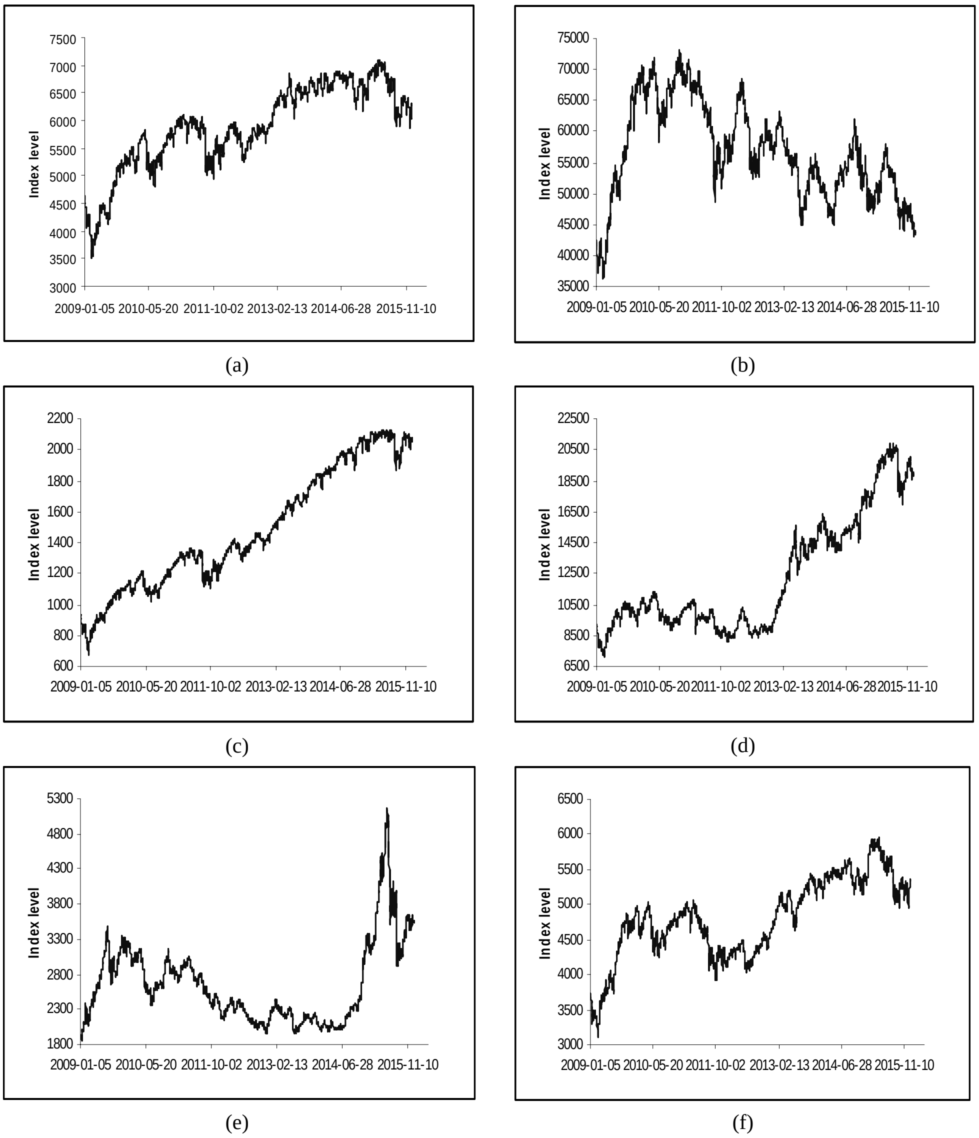
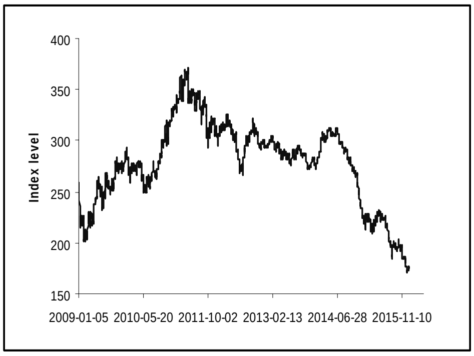
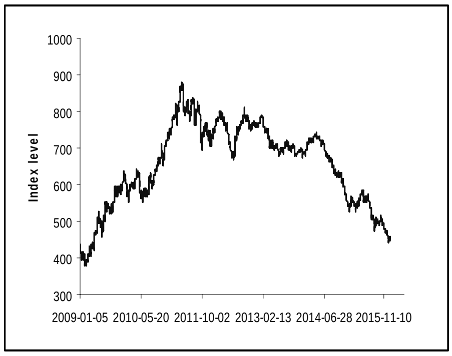


Fig. 1. Stock indexes from January 5, 2009 to December 30, 2015: FTSE 100 (a), IBOVESPA (b), S&P 500 (c), Nikkei 225 (d), SSE (e), AOI (f)

Source: own elaboration.



(a)



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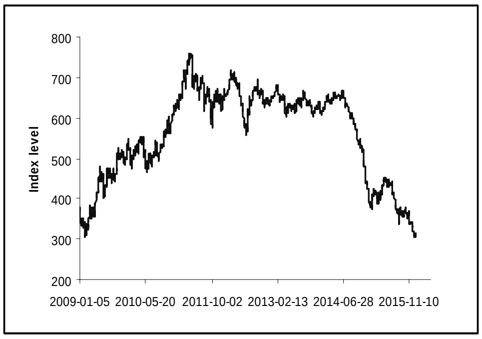

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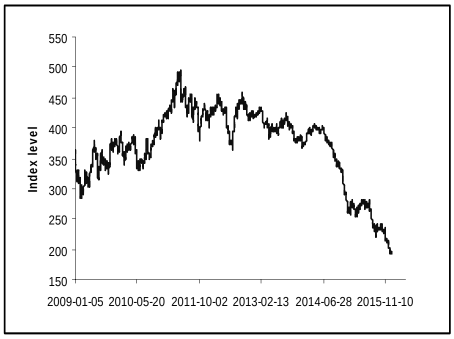

(d)

Fig. 2. Commodity indexes from January 5, 2009 to December 30, 2015: the Reuters/ Core Commodity CRB Index (a), Bloomberg Commodity Index (b), S&P Goldman Sachs Commodity Index (c), Deutsche Bank Liquid Commodity Index (d)

Source: own elaboration.

Table 2. Mean returns for commodity and stock indexes

Index	Year							
	2009	2010	2011	2012	2013	2014	2015	2009-15
TR/CC CRB	0.00047	0.00062	-0.00033	-0.00013	-0.00020	-0.00077	-0.00107	-0.00020
BCOM	0.00141	0.00077	-0.00007	0.00015	-0.00036	-0.00082	-0.00084	0.00003
S&P GSCI	0.00135	0.00072	0.00008	0.00001	-0.00009	-0.00162	-0.00118	-0.00010
DBLCI	0.00022	0.00045	-0.00004	0.00003	-0.00039	-0.00120	-0.00146	-0.00034
FTSE 100	0.00065	0.00033	-0.00022	0.00022	0.00052	-0.00011	-0.00018	0.00018
IBOVESPA	0.00200	0.00002	-0.00076	0.00029	-0.00075	-0.00013	-0.00050	0.00002
S&P 500	0.00072	0.00046	0.00000	0.00049	0.00101	0.00042	0.00001	0.00044
Nikkei 225	0.00062	-0.00012	-0.00076	0.00077	0.00175	0.00032	0.00031	0.00041
SSE	0.00217	-0.00060	-0.00095	0.00012	-0.00027	0.00166	0.00035	0.00035
AOI	0.00115	-0.00003	-0.00064	0.00049	0.00053	0.00003	-0.00003	0.00021

Source: own calculations.

Table 3. Standard deviations for commodity and stock indexes logarithmic returns

Index	Year							
	2009	2010	2011	2012	2013	2014	2015	2009-15
TR/CC CRB	0.01636	0.01042	0.01159	0.00880	0.00580	0.00645	0.01103	0.01058
BCOM	0.01530	0.01115	0.01174	0.00840	0.00607	0.00596	0.00961	0.01023
S&P GSCI	0.02028	0.01282	0.01429	0.01052	0.00731	0.00832	0.01503	0.01331
DBLCI	0.01778	0.01245	0.01381	0.01045	0.00747	0.00787	0.01321	0.01232
FTSE 100	0.01449	0.01078	0.01316	0.00844	0.00753	0.00711	0.01066	0.01062
IBOVESPA	0.01860	0.01241	0.01530	0.01311	0.01255	0.01538	0.01394	0.01460
S&P 500	0.01657	0.01114	0.01450	0.00783	0.00686	0.00711	0.00958	0.01106
Nikkei 225	0.01662	0.01260	0.01428	0.00990	0.01652	0.01218	0.01285	0.01374
SSE	0.01868	0.01380	0.01137	0.01060	0.01116	0.01097	0.02388	0.01510
AOI	0.01255	0.00948	0.01181	0.00719	0.00753	0.00684	0.01016	0.00960

Source: own calculations.

Results in table 3 show that the least risky asset is the AOI exhibiting the lowest standard deviation values in 2009, 2010 and 2012 as well as in the whole period under consideration (2009-2015). The highest standard deviation in the whole period as well as in 2010 and in 2015 is generated by the Shanghai Composite Index (SSE) which makes it the riskiest one among investigated indexes.

In table 4, there are presented values of Pearson correlation coefficient. Obviously, commodity indexes returns reveal strong linear dependences (the strongest positive correlation among commodity indexes is the one between S&P GSCI and DBLCI). Although previous studies often reported negative correlations between commodity and stock returns (Greer, 2000; Gorton and Rouwenhorst, 2004; Schofield, 2007; Chevalier and Ielpo, 2013), we do not find such relations in our data. All commodity and stock indexes are characterized by positive linear correlation, however the weakest dependencies are between commodity indexes and the Nikkei 225.

Table 4. Correlation coefficients between commodity and stock indexes logarithmic returns

Index	TR/CC CRB	BCOM	S&P GSCI	DBLCI	FTSE 100	IBOV ESPA	S&P 500	Nikkei 225	SSE	AOI
TR/CC CRB	1									
BCOM	0.918	1								
S&P GSCI	0.916	0.917	1							
DBLCI	0.915	0.930	0.965	1						
FTSE 100	0.467	0.505	0.470	0.444	1					
IBOVESPA	0.446	0.441	0.436	0.421	0.519	1				
S&P 500	0.474	0.475	0.485	0.466	0.651	0.625	1			
Nikkei 225	0.146	0.126	0.112	0.094	0.257	0.115	0.137	1		
SSE	0.183	0.203	0.150	0.151	0.183	0.161	0.113	0.262	1	
AOI	0.190	0.194	0.172	0.149	0.324	0.206	0.238	0.470	0.224	1

Source: own calculations.

Empirical results

In the first step of the research, for the purpose of examining portfolio performance and stability over time, the data is divided into seven annual periods. Employing the two options of Markowitz model, we receive optimal portfolios for each of the periods, individually. Tables 5 and 6 present the composition of optimal portfolios.

Table 5. Composition of Markowitz optimal portfolios (%) – option A

Index	Year						
	2009	2010	2011	2012	2013	2014	2015
TR/CC CRB	0.00	23.18	28.12	0.00	49.21	12.60	0.00
BCOM	20.43	0.00	0.00	19.04	0.00	32.78	42.66
S&P GSCI	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DBLCI	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FTSE 100	19.25	12.29	1.49	9.24	13.92	10.23	6.05
IBOVESPA	0.00	7.14	0.00	0.00	1.09	0.00	0.00
S&P 500	14.20	17.92	16.17	32.63	22.19	20.97	28.68
Nikkei 225	27.97	24.75	13.67	20.08	3.61	7.34	22.38
SSE	17.80	14.36	40.39	18.80	9.86	15.93	0.00
AOI	0.36	0.37	0.16	0.21	0.12	0.15	0.23
Total	100	100	100	100	100	100	100

Source: own calculations.

Table 6. Composition of Markowitz optimal portfolios (%) – option B*

Index	Year						
	2009	2010	2011	2012	2013	2014	2015
TR/CC CRB	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BCOM	0.15	66.93	63.79	0.00	0.00	0.00	0.00
S&P GSCI	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DBLCI	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FTSE 100	0.00	1.44	0.00	0.00	0.00	0.00	0.00
IBOVESPA	45.83	0.00	0.00	0.00	0.00	0.00	0.00
S&P 500	0.00	25.63	32.94	7.72	31.49	0.88	0.00
Nikkei 225	0.00	5.94	1.59	92.28	68.51	0.00	38.02
SSE	53.78	0.00	1.57	0.00	0.00	99.12	61.98
AOI	0.25	0.05	0.11	0.00	0.00	0.00	0.00
Total	100	100	100	100	100	100	100

*The predetermined maximal portfolio variance of returns is set to 75% of $\max S_i^2$. Relaxing this constraint results in the limitation of portfolio composition to one or maximally two assets.

Source: own calculations.

The results in table 5 show that portfolios' component assets, as well as their weights, generally change from year to year with the exception of the S&P GSCI and DBLCI. They are not components of any of the portfolios. The shares of commodities represented by commodity indexes range between 19.04% in 2012 and 49.21% in 2013. Most of the component assets for year *t* remain in the optimal portfolio for year *t*+1, although the weightings can vary considerably.

The results in Table 6 also show the instability of portfolios' components and their weights with exception of the TR/CC CRB Index, S&P GSCI and DBLCI. Their weights are equal to zero regardless the year under consideration. This time commodities play less important role in optimal portfolios. The BCOM is the only one of commodity indexes in optimal structures. However, it appears as portfolios' component only in 2009 and in 2010. In general, the number of components in portfolios of option B is lower than the number of components in portfolios of option A.

In practice, investors have to determine the portfolio weights at time (*t*) for the subsequent period [*t*, *t*+1] using only data available at time (*t*). It would be interesting to compare the ex-post results to the equally weighted portfolios, then. Thus, in the next step of the research, in order to examine the resultant ex-post "optimal" portfolios at time (*t*+1), we apply the optimal weights at year (*t*) to the data in year (*t*+1), e.g. the component weights of the optimal portfolio from 2009 are applied to data from 2010, etc. The return and risk of optimal and equally weighted portfolios are presented in tables 7 and 8.

Table 7. Return and risk of optimal and equally weighted portfolios (option A)

Year	Optimal portfolios		Equally weighted portfolios	
	Mean return	Standard deviation	Mean return	Standard deviation
2010	0.00015	0.00841	0.00026	0.01295
2011	-0.00049	0.00901	-0.00037	0.01431
2012	0.00020	0.00658	0.00024	0.01071
2013	0.00061	0.00592	0.00018	0.01004
2014	-0.00013	0.00427	-0.00022	0.00937
2015	-0.00038	0.00823	-0.00049	0.01289

Source: own calculations.

Table 8. Return and risk of optimal and equally weighted portfolios (option B)

Year	Optimal portfolios		Equally weighted portfolios	
	Mean return	Standard deviation	Mean return	Standard deviation
2010	-0.00031	0.01060	0.00026	0.01295
2011	-0.00009	0.01065	-0.00037	0.01431
2012	0.00027	0.00717	0.00024	0.01071
2013	0.00170	0.01529	0.00018	0.01004
2014	0.00035	0.00887	-0.00022	0.00937
2015	0.00032	0.02365	-0.00049	0.01289

Source: own calculations.

Investors should prefer optimal Markowitz portfolios only if they provide return and risk diversification benefits that are superior to equally weighted portfolios. The results in table 7 show that in 2013, 2014 and 2015 "optimal" ex-post portfolios (option A) outperform equally weighted portfolios, reducing the risk of investing and enhancing the profits. In other cases they provide lower standard deviations (lower risk), however combined with lower profit. The results in table 8 show that "optimal" ex-post portfolios (option B) outperform equally weighted portfolios in 2011, 2012 and 2013. In other cases they offer higher profits combined with higher standard deviations (with the exception of 2010). Summing up the results for options A and B, we can conclude that 2013 is the only year when "optimal" ex-post portfolios outperform the equally weighted one, regardless the option of Markowitz optimization model.

Concluding remarks

Since early 2000s, commodities have emerged as an additional asset class alongside with traditional ones as stocks and bonds. Slightly negative return correlations between commodity and stock returns imply an opportunity for diversification and thus attracts investors. This growing interest results in number of papers examining diversification benefits of adding to portfolio of stocks or bonds different commodity investment vehicles, such as commodity-related stocks, commodity futures or commodity indexes. Studies on commodity diversification typically focus on the risk and/or return benefits of adding a commodity to a portfolio of stocks and bonds and they often employ an equally weighted portfolio instead of generating optimal portfolios. Moreover, they usually ignore the stability of optimal portfolio weights over time and the performance of optimal portfolio in subsequent periods of time.

In this study we examine the stability of the weights of the components in the Markowitz portfolio composed of commodity and stock indexes. Optimal portfolios are set by employing two options of Markowitz model: one that minimizes portfolio variance of returns and the other one that maximizes portfolio return. When we examine the components of the Markowitz portfolios, we find that portfolio composition and weights are not stable over time. Thus, if the stability of the component assets is the primary goal, then equally weighted portfolio is preferable. Comparing equally weighted and optimal portfolios' risk and return characteristics, we find optimal portfolios outperforming equally weighted ones in half of analyzed cases. Our results are in general consistent with findings of You and Deigler (2012), however they examine the diversification benefits of using individual futures contracts instead of commodity indexes.

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